Fundamentals of Information Theory II

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Reading

- http://mathworld.wolfram.com/SamplingTheorem.html
- http://cwx.prenhall.com/bookbind/pubbooks/cyganski/chapter0/deluxe.html - demo applets

Representing signals: frequency, time, amplitude

- Baseband signals: $F(t)$ limited in frequency in the interval $[-W, W]$.
- Broadband signals (carrier based):

  Moved in the interval $[W_c - W, W_c + W]$

  Example: $F(t) = F_s(t)\sin(W_c t)$

Representing signals by sample points

- Theoretically, infinite number of points are required.
- Locating a point in space: coordinate systems.
- Locating a function $F(t)$:

  vector $F(t_1), F(t_2), ..., F(t_n)$

  t_n - sample points
- Orthonormal sets of functions: $\{G_n(X)\}$, where:

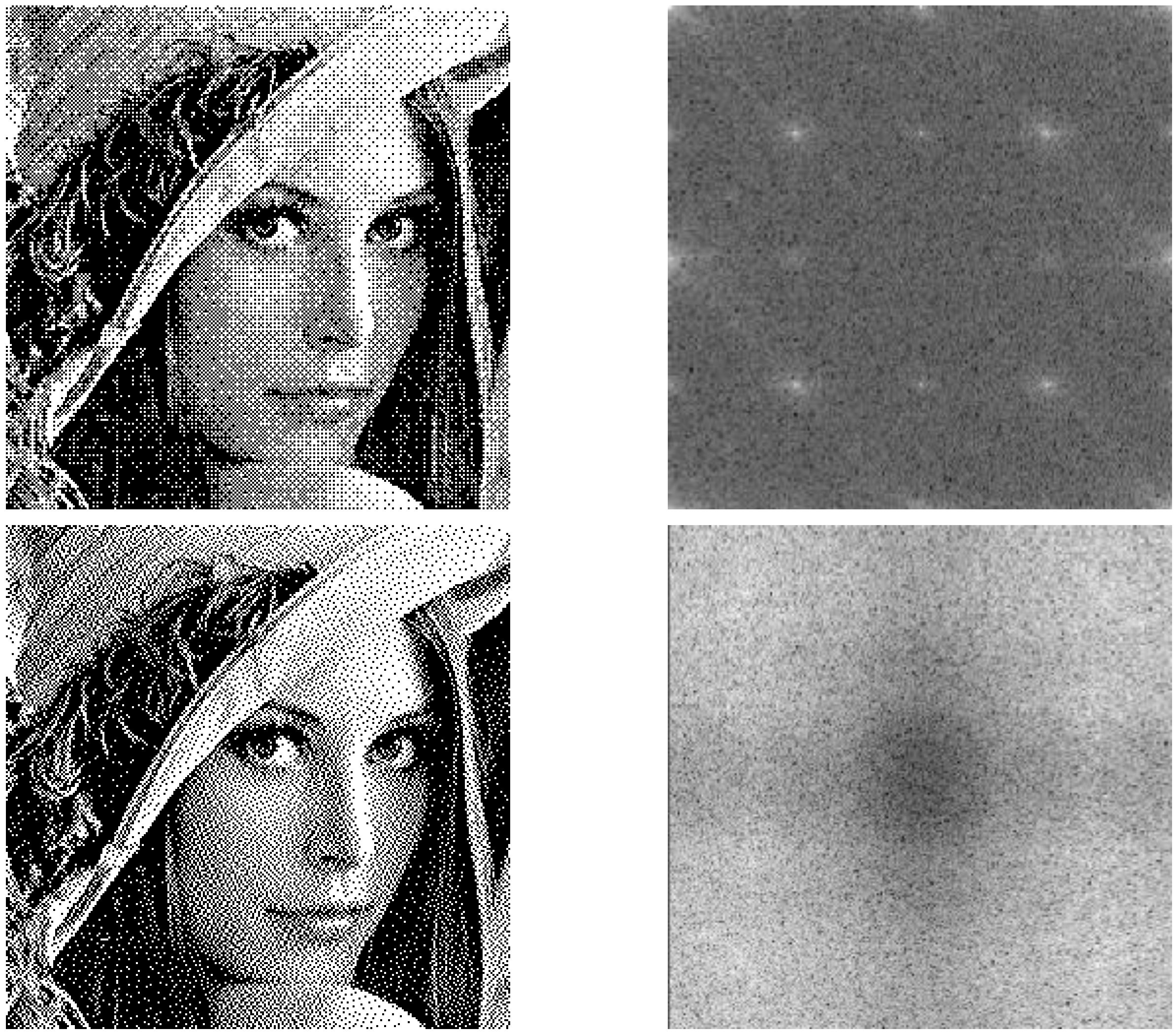
$$\int_a^b G_n(x)G_m(x)dx = \begin{cases} 1 & \text{if } n = m; \\ 0 & \text{if } n \neq m. \end{cases}$$

- Expansion of a function (Fourier series):

$$F(x) = \sum_{n=-\inf}^{n=+\inf} C_n G_n(x)$$

$$C_m = \int_{-\inf}^{+\inf} F(x)G_m(x)dx$$

Displaying 2D Magnitude Spectra of Quantization Noise



Sampling theorem

- Orthonormal set: $\{sinc_n(t)\}$, $t \in [-T, +T]$, $T \rightarrow \inf$.

$$sinc_n(t) = \frac{\sin(2\pi W(t - \frac{n}{2W}))}{2\pi W(t - \frac{n}{2W})}$$

- Fourier expansion of $F(t)$:

$$F(t) = \sum_{n=-\inf}^{n=+\inf} C_n \frac{\sin(2\pi W(t - \frac{n}{2W}))}{2\pi W(t - \frac{n}{2W})}$$

- Sample points: ...,$t_1, t_2, ..., t_n$,..., where $t_n = \frac{n}{2W}$, $n \in [-\inf, +\inf]$ and $[-W, +W]$ is the frequency interval.
- Let $t = t_n$, for $n \in [-\inf, +\inf]$. Then all terms in the Fourier expansion of $F(t)$ above are 0 except one, $\frac{\sin(0)}{0} = 1$. That is, $F(t_n) = C_n$.
- Hence (sampling theorem):

$$F(t) = \sum_{n=-\inf}^{n=+\inf} f(t_n) \frac{\sin(2\pi W(t - \frac{n}{2W}))}{2\pi W(t - \frac{n}{2W})}$$

- Constraints:

  $F(t)$ is limited in frequency in $[-W, W]$,

  There are infinite number of sample points (infinite time duration)

AD and DA conversion

- quantization (thresholds)
- amplifiers vs. repeaters
- generally very low error rates (regenerating digital values at relay points)

Error detection and correction

- Binary case: 000,111 for 0,1 - one error correcting code
- Arbitrary waveform: three instead of one sample point - widening the bandwidth (2TW dimensions)