Fundamentals of Information Theory I

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Fall 2007

CS 502: Computers and Communications Technology

Lecture 17, November 5, 2007

Reading

- http://cm.bell-labs.com/cm/ms/what/shannonday/paper.html

Communication System

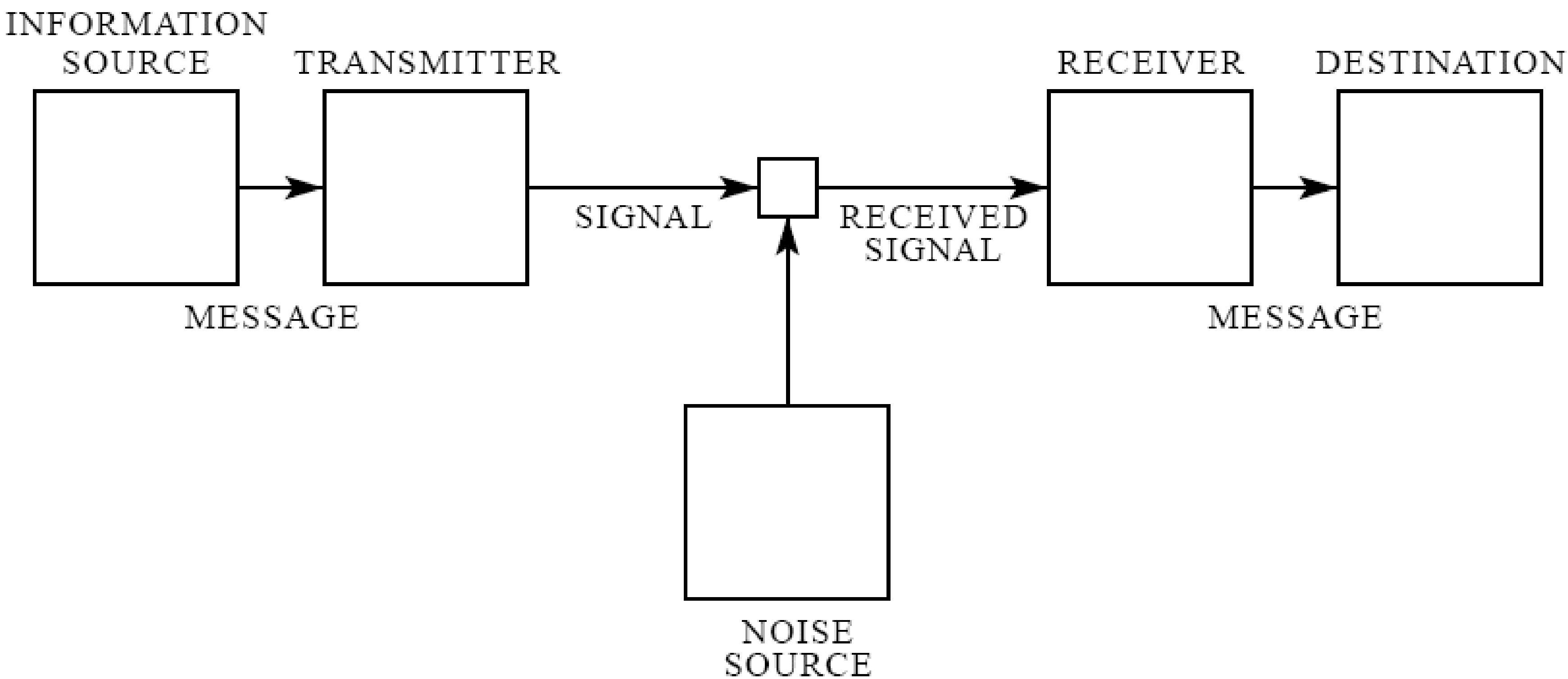

Fig. 1—Schematic diagram of a general communication system.

Additivity on the Logarithmic Scale

- $P(A \cdot B) = P(A) \cdot P(B)$ if A and B are independent random events
- $\log_2 P(A \cdot B) = \log_2 P(A) + \log_2 P(B)$
- However, as $0 \leq P(E) \leq 1$,

  $-\infty \leq \log_2 P(E) \leq 0$, so it is more convenient to use…

Entropy

$$H = -\sum_{i=1}^{n} p_i \log p_i$$

Entropy H is a measure of uncertainty.

Example 1

- Two possibilities with probabilities *p* and *q=1-p*,
  $H = -(p \cdot \log_2 p + q \cdot \log_2 q)$

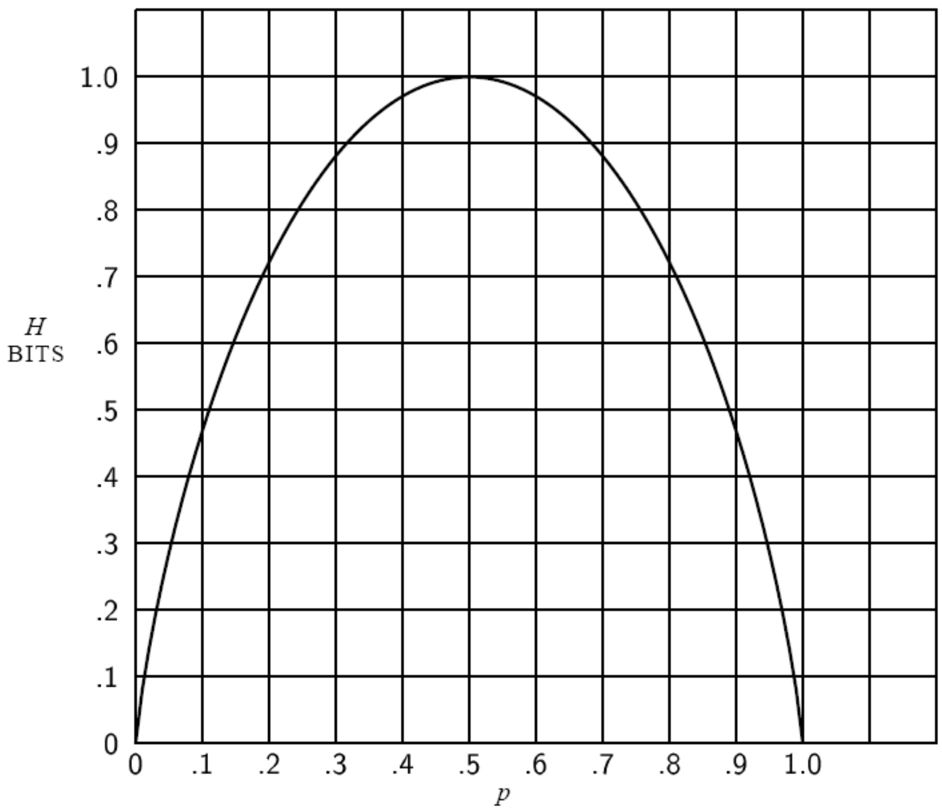

Fig. 7—Entropy in the case of two possibilities with probabilities p and $(1-p)$.

Example 2

- $X=\{0,1,\ldots,7\}$,
  $P(X=1)=P(X=2)=...=P(X=7)=1/8$;
  $H(X)=-8*(1/8)*log(1/8)=3$ (bits)

Properties of Entropy

1. H is continuous in the p_i.
2. If all the p_i are equal, $p_i=1/n$, then H is a monotonic increasing function of n.
3. The original H should be the weighted sum of the individual values,

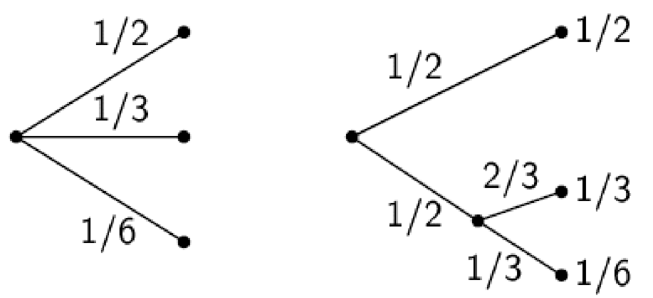

Fig. 6—Decomposition of a choice from three possibilities.

H(1/2,1/3,1/6) = H(1/2,1/2) + 0.5*H(2/3,1/3)

Properties of Entropy (cont'd)

4. $H=0$ if and only if all the p_i but one are zero
5. For a given n, H is a maximum and equal to $\log_2 n$ when all p_i are equal $1/n$.
6. The uncertainty H of a joint event is less than or equal to the sum of the individual uncertainties, with equality only if the individual events are independent.
7. The uncertainty of one event is never increased by knowledge of another event. It will be decreased unless the two events are independent.

Data Compression

Save storage space; speed up transmission.

Bandwidth: Bits (bytes) per second

Compression ratio: $\dfrac{size\_of\_the\_compressed\_data}{size\_of\_the\_uncompressed\_data}$

Lossless vs. *lossy* compression

Keyword encoding: Replace a popular word with a shorter code (“with” → “w/”, “without” → “w/o”)

Run-length encoding: AAAAAA → A6

Can combine the two.

Huffman Encoding