Components of Computing Systems

• Hardware: Circuit boards, chips, disk drives, peripherals, wires, etc.
• Software: Programs (sequences of instructions for the computer to carry out)
• Data (information in its digital form)
Layers of a Computing System

Information

Hardware

Operating systems

Applications

Communications

Programming:
- systems programming
- applications programming
Abstraction…

...removes or hides complex details.
The History of Computing

• Textbook, Section 1.5
• http://www.computerhistory.org/exhibits/internet_history/
Layers of Software

Application packages

High-level languages

Assembly languages

Systems software

Machine languages
Number Categories

- **Natural numbers**: The number 0 and numbers obtained by repeatedly adding 1 to this number. Example: \(3 = 0 + 1 + 1 + 1\)

- **Negative numbers**: Less than 0. Example: \(-\sqrt{2}\)

- **Integers**: Natural numbers and their negatives

- **Rational numbers**: Fractions, quotients of two integers. Examples: \(16/13; 4/1=4\)

- **Irrational numbers**: Cannot be represented as quotients of two integers. Example: \(\sqrt{2}\)
How to represent a natural number?

*Base* of a number system: The number of digits used in the system. Example 1: Base 10 (*decimal*)

\[1760_{10} = 0 \times 10^0 + 6 \times 10^1 + 7 \times 10^2 + 1 \times 10^3\]

Numbers are written using *positional notation*.

Example 2: Base 2 (*binary*)

\[11101_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 = 29_{10}\]
More Number Systems!

Example 3: *Octal* (Base 8)

\[ 73_8 = 3 \times 8^0 + 7 \times 8^1 = 59_{10} = 111011_2 \]

Example 4: *Hexadecimal* (Base 16)

\[ AF_{16} = 15 \times 16^0 + 10 \times 16^1 = 175_{10} = 257_8 = 10101111_2 \]

Extra digits: A=10, B=11, C=12, D=13, E=14, F=15
How to represent a ratio?

259:160=1.61875  241:149≈1.61745

A ratio is represented by an angle here.
How to represent an irrational number?

1. Geometrically

\[ \sqrt{2} \]

2. By an algorithm: The Fibonacci numbers algorithm is a way to represent the golden ratio

\[ \phi = \frac{1 + \sqrt{5}}{2} \approx 1.618034 \]
Addition and Subtraction in Binary

1 11 ←carry  1 ←carry

10011 → 1+2+16 = 19
11001 + 1+8+16 = +25
101100 → 4+8+32 = 44

1 1 ←borrow

10101 1+4+16 = 21
10111 - 1+2+ 8 = −11
1010 2+8 = 10
Power of Two Number Systems

1 digit in Base 8 = $2^3$ (octal) corresponds to 3 digits in Base 2 (binary):

$0_8 = 000_2$

Example of conversion:

$11001110_2 = (011)(001)(110) = 316_8$

$2_8 = 010_2$

Indeed, $2 + 4 + 8 + 64 + 128 = 206_{10}$ and

$3_8 = 011_2$

$6 + 1 \times 8 + 3 \times 8^2 = 206_{10}$

$4_8 = 100_2$

$5_8 = 101_2$

$6_8 = 110_2$

$7_8 = 111_2$
Power of Two Number Systems (cont’d)

1 digit in Base 16 = 2^4 (hexadecimal) corresponds to 4 digits in Base 2 (binary):

Example of conversion:

11001110_2 = CE_{16}

<table>
<thead>
<tr>
<th>Base 16</th>
<th>Base 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000_2</td>
</tr>
<tr>
<td>1</td>
<td>0001_2</td>
</tr>
<tr>
<td>2</td>
<td>0010_2</td>
</tr>
<tr>
<td>3</td>
<td>0011_2</td>
</tr>
<tr>
<td>4</td>
<td>0100_2</td>
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<td>5</td>
<td>0101_2</td>
</tr>
<tr>
<td>6</td>
<td>0110_2</td>
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<td>7</td>
<td>0111_2</td>
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<td>8</td>
<td>1000_2</td>
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<tr>
<td>9</td>
<td>1001_2</td>
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<td>1010_2</td>
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<tr>
<td>11</td>
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<tr>
<td>12</td>
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<tr>
<td>13</td>
<td>1101_2</td>
</tr>
<tr>
<td>14</td>
<td>1110_2</td>
</tr>
<tr>
<td>15</td>
<td>1111_2</td>
</tr>
</tbody>
</table>
Converting from Base 10 to Other Bases

Converting $2849_{10}$ to hexadecimal (Base 16):

$2849/16=178.0625$; $178.0625-178=0.0625$;
$0.0625*16=1$, so 1 is the first digit from the right.
$178/16=11.125$; $11.125-11=0.125$;
$0.125*16=2$, so 2 is the second digit.
$11<16$, so B is the third and the last digit.

Indeed,

$B21_{16} = 1+2*16+11*16^2 = 2849_{10}$