Version space learning

Language for examples/hypotheses

\[ L = \{ [A, B], A \in T_1, B \in T_2 \}. \]

- **\( T_1 \)** – taxonomy of colors
- **\( T_2 \)** – taxonomy of planar geometric shapes.

```
any_color
________|________
|     |     |
mono  poly
________|________  ___|
|     |     |     |
red  blue  white  black  orange  pink
```

```
any_shape
-----------------|-----------------
|                 |
polygon          oval
-----------------|-----------------
|     |     |     |
3-sided  4-sided
|     |     |     |
|     |     |     |
triangle  rectangle  square  trapezoid  circle  ellipse
```
Ordering relations

\[ [A_1, B_1] \geq [A_2, B_2], \] if \( A_2 \) is a successor of \( A_1 \) in \( T_1 \), and \( B_2 \) is a successor of \( B_1 \) in \( T_2 \).

\[ [\text{red, polygon}] \geq [\text{red, triangle}] \]
\[ [\text{any\_color, any\_shape}] \text{ covers all possible examples in } L \text{ (how many?)} \]

Induction task

Given:
\( E_1^+ = [\text{red, square}] \)
\( E_2^+ = [\text{blue, rectangle}] \)
\( E_3^- = [\text{orange, triangle}] \)

Find \( H \), such that:
\( H \geq E_1^+, H \geq E_2^+ \)
\( H \nless E^- \)

Hypothesis space

- Generate all generalizations of \( E_1^+, E_2^+ \). \( S_H = \{[\text{mono, 4\_sided}], 
[\text{mono, polygon}], \ldots, [\text{any\_color, any\_shape}]\} \).
- Remove from \( S_H \) all hypotheses that cover \( E_3^- \). Version space, \( VS = \{[\text{mono, 4\_sided}], [\text{mono, polygon}], [\text{mono, any\_shape}], [\text{any\_color, 4\_sided}]\} \).
Specific to general search (Find-S)

Maximally specific generalizations $S$: $H \in S$, if $H \in VS$ and for any $H' \in VS$, $H' \geq H$.

Begin Find-S

Initialize $S$ to the first positive example

Initialize $N$ to all negative examples seen so far

For each positive example $E^+$ do begin

Replace every $H \in S$, such that $H \nsubseteq E^+$, with all its generalizations that cover $E^+$

Delete from $S$ all hypotheses that cover other hypotheses in $S$

Delete from $S$ all hypotheses that cover any element from $N$

End

For every negative example $E^-$ do begin

Delete all members of $S$ that cover $E^-$

Add $E^-$ to $N$

End

End

End Find-S
General to specific search (Find-G)

Maximally general hypotheses $G$: $H \in G$ if it covers none of the negative examples, and for any other hypothesis $H'$ that covers no negative examples, $H \geq H'$.

Begin Find-G

Initialize $G$ to the most general concept in the version space

Initialize $P$ to all positive examples seen so far

For each negative example $E^-$ do begin

Replace every $H \in G$, such that $H \geq E^-$, with all its specializations that do not cover $E^-$

Delete from $G$ all hypotheses more specific (covered by) other hypotheses in $G$

Delete from $G$ all hypotheses that fail to cover some example from $P$

End

For every positive example $E^+$ do begin

Delete all members of $G$ that fail to cover $E^+$

Add $E^+$ to $P$

End

End

End Find-G
Combining Find-$S$ and Find-$G$

Boundary set theorem (Genesereth and Nilsson, 1987): For every $H \in V_S$, there exist $H' \in S$ and $H'' \in G$, such that $H \geq H'$ and $H'' \geq H$.

Candidate elimination algorithm (Mitchel, 82)

Put together Find-$S$ and Find-$G$ and:

- Replace ”Delete from $S$ all hypotheses that cover any element from $N$” with ”Delete from $S$ any hypothesis not more specific than some hypothesis in $G$”

- Replace ”Delete from $G$ all hypotheses that fail to cover some example from $P$” with ”Delete from $G$ any hypothesis more specific than some hypothesis in $S$”

- No need of maintaining $P$ and $N$.

Stopping conditions

- If $G = S$ and both are singletons, then stop. The algorithm has found a single hypothesis consistent with the examples.

- If $G$ or $S$ becomes empty then stop. Indicate that there is no hypothesis that covers all positive and none of the negative examples.
Experiment Generation, Interactive Learning

1. Ask for the first positive example
2. Calculate $S$ and $G$ using the candidate elimination algorithm
3. Find $E$, such that $G \geq E, \forall s \in S, E \not\geq s$ ($E$ is not in the version space).
4. Ask about the classification of $E$
5. Go to 2